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Walsh-Hadamard transformation of television pictures

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Summary

This report gives a summary of the basic theoretical background of Walsh-Hadamard transformation and describes a method by which the process may efficiently be carried out on real-time television signals. The use of such a transformer, augmented by signal measurements carried out in non-real time, is described. The use of the technique as a means of television bit-rate reduction is discussed.

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1. Introduction

High quality television pictures require a very high data-rate for adequate representation when coded by a linear p.c.m. system. It has been suggested that some form of transform picture coding method would reduce the data rate without significantly impairing the picture quality, by exploiting certain redundancies within the television signal. These redundancies are not normally accessible, but become accessible in the transform domain.

The only feasible transform, at the high data rate required, is the Hadamard. A Hadamard matrix is an orthogonal matrix in which all of the elements take the values +1 or −1. The calculations required to do the transformation are thus reduced to addition and subtraction, replacing the multiplications normally required, for example, in discrete Fourier transformation. One particular sub-set of Hadamard matrices is the Walsh-Hadamard for which there exists a fast transform algorithm, which reduces the amount of calculation required.

The impairments which would result from a number of storage and arithmetic operations on an analogue signal preclude the use of such methods for all but the smallest transforms, if picture quality is to be maintained. Digital methods are not subject to these impairments and in addition, the ready availability and low cost of suitable circuit-elements, as a result of the mass production required by the computer industry, make the digital approach feasible, even for large orders of transform.

Two approaches to an investigation of digital Walsh-Hadamard transformation and the subsequent bit-rate reduction are possible. One approach is to construct a real-time transformer, thereby enabling normal picture/programme material to be assessed under normal viewing conditions. This requires special-purpose processing equipment operating at very high speeds. Such equipment is expensive and not easy to modify to different transform orders, even if constructed in the modular way made possible by the structure of the fast transform algorithm.

The second approach is to construct a Walsh spectrum analyser (non-real time) to be used for analysing the Walsh spectrum of real television pictures, that is, the distribution of the energy in the transform domain. This type of equipment does not work in real time, but it has the advantages of being relatively cheap and readily adapted to analyse different transform orders.

The two approaches are complementary and, by using the results obtained by spectral analysis, the observed effects on the pictures obtained using the real-time transformer can be extrapolated.

An investigation into the application of Hadamard transformation to digital television bit-rate reduction is nearing completion and will be fully described in three Research Department reports.[1,2,3] This report presents the main aspects in summarised form and gives some of the results so far obtained.

2. Theory of Hadamard transformation

Hadamard transformation is defined by the matrix equation:

$$\begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_n \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \ldots h_{1n} \\ h_{21} & h_{22} \ldots h_{2n} \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ h_{n1} & h_{n2} \ldots h_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \qquad (1)$$

or $\bar{c} = H\bar{x}$

where $\bar{x}$ and $\bar{c}$ represent input and output n-dimensional vectors and H represents a Hadamard matrix of order n. A Hadamard matrix has two properties: it is orthogonal, that is:

$$HH^T = nI \qquad (2)$$

where H^T represents the transpose of H and I represents the unit matrix of order n; and the elements of the matrix may take only the values +1 or −1, that is:

$$h_{ij} = \pm 1 \qquad i = 1,n; \; j = 1,n.$$

Some Hadamard matrices are symmetric, that is:

$$H = H^T$$

so that Equation (2) becomes

$$HH = nI \qquad (3)$$

Then, in the inverse transformation,

$$\bar{x} = H^{-1}\bar{c}$$

the inverse Hadamard matrix H^{-1} is given by:

$$HHH^{-1} = nH^{-1} \qquad \text{using (3)}$$

therefore $$H^{-1} = \frac{1}{n}H \qquad (4)$$

and
$$\overline{x} = \frac{1}{n} H \overline{c} \tag{5}$$

So, for symmetric Hadamard matrices, the inverse transformation is the same as the forward transformation except that it includes division by the constant n.

A particular set of symmetric Hadamard matrices, known as Walsh-Hadamard matrices, is formed by the direct products of the matrix H_2 with itself, that is:

$$H_2p = H_2^{(1)} \otimes H_2^{(2)} \otimes \ldots \otimes H_2^{(p)}$$

where
$$H_2^{(r)} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad r = 1,p$$

and H_2p is the Walsh-Hadamard matrix of order 2^p. The direct or Kronecker product operation,[4] denoted $\otimes$, is defined for a (u x u) matrix A and a (v x v) matrix B as:

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \ldots a_{1u}B \\ a_{21}B & a_{22}B \ldots a_{2u}B \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ a_{u1}B & a_{u2}B \ldots a_{uu}B \end{bmatrix}$$

where A $\otimes$ B is a (uv x uv) matrix. So Walsh-Hadamard matrices of order $n = 2^p$ can be constructed where p is a positive integer.

Equation (1) may be re-written as:

$$c_i = \sum_{j=1}^{n} h_{ij} x_j \qquad i = 1,n \tag{6}$$

which shows c_i as a measure of the correlation between the input vector $\overline{x}$ and a discrete two-valued function defined by the ith row of a Hadamard matrix. The functions formed by the rows of the Walsh-Hadamard matrix of order n are the set of Walsh functions of period n. So, the vector of coefficients produced by Walsh-Hadamard transformation describes the input data in terms of the amplitudes of a set of Walsh functions.

Fourier analysis of Walsh functions shows that, to a first approximation, there is a linear relationship between the sequency of each Walsh function (the number of transitions between +1 and −1 in a period) and the frequency of the largest sinusoidal component of the Walsh function. For this reason, in the Walsh-Hadamard transform or sequency spectrum of a signal, the low sequency components tend to represent low frequencies in the signal and the high sequency components tend to represent high frequencies in the signal.

3. Fast transform

3.1. The fast transform algorithm

By using a fast algorithm, the Walsh-Hadamard transform can be calculated more efficiently than by the direct matrix multiplication discussed in the previous section. In the fast transform method, the input vector is multiplied by a number of sparse matrices (matrices with a small number of non-zero elements), the product of which is the Walsh-Hadamard matrix. For example, the Walsh-Hadamard matrix of order 8 can be expressed as a product of three sparse matrices S_1, S_2 and S_3, such that:

$$H_8 = S_3 S_2 S_1 \tag{7}$$

where
$$S_1 = \begin{bmatrix} +1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} +1 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & +1 & 0 & 0 & 0 & 0 \\ +1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 \\ 0 & 0 & 0 & 0 & +1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & -1 \end{bmatrix}$$

and
$$S_3 = \begin{bmatrix} +1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 \end{bmatrix}$$

The transform is calculated by the following three products of an (8 x 8) matrix and an eight-element vector:

$$\begin{aligned} \overline{a} &= S_1 \overline{x} \\ \overline{b} &= S_2 \overline{a} \\ \overline{c} &= S_3 \overline{b} \end{aligned} \tag{8}$$

where $\overline{a}$ and $\overline{b}$ are eight-element vectors produced after the first and second stages of the transformation, and $\overline{c}$ is the vector of transform coefficients. Since each of the sparse matrices has only two non-zero elements in each row, each of the products of a matrix and vector requires only eight additions or subtractions. In general, an (n x n) Walsh-Hadamard matrix can be expressed as a product of $\log_2 n$ sparse matrices and each product of a sparse matrix and a vector requires n additions or subtractions. Thus the transform can be calculated by the fast algorithm in $n \log_2 n$ operations. For the same order of matrix, the direct multiplication method requires (n − 1) additions or subtractions to calculate each of the n coefficients, so that the transform is calculated in n(n − 1) operations.

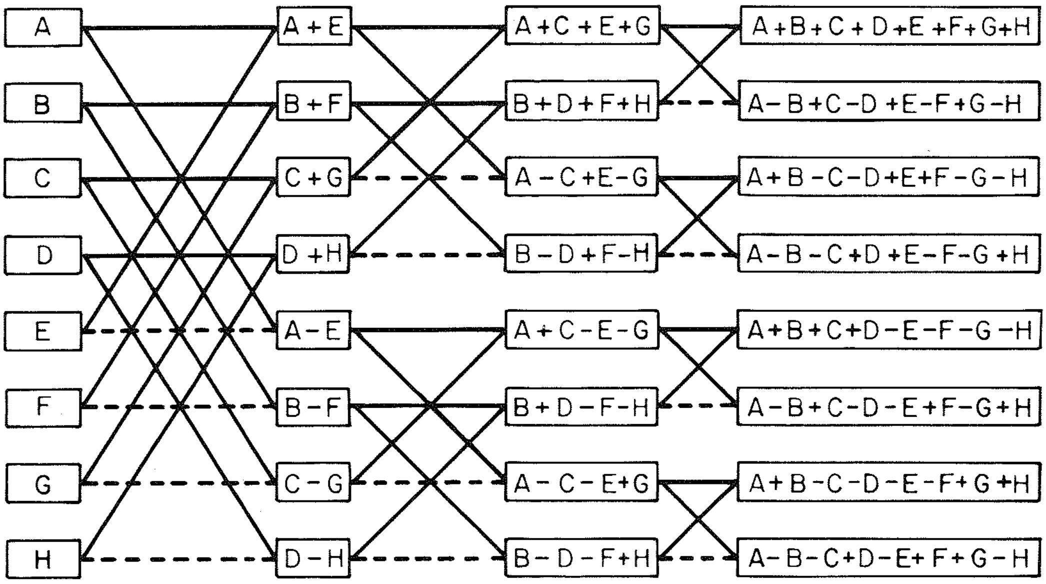

Fig. 1 - Signal flow diagram for fast Walsh-Hadamard transform of order 8

3.2. Real-time transformer

The operations defined by the sparse matrices above can be represented by a signal flow diagram such as that shown in Fig. 1[5] for a transform of order 8. This diagram may be interpreted as a programme for retrieving information from storage locations in order to perform the transformation. It can be seen that each element of the input vector and each element of the intermediate results are stored only once. It is also apparent that the results at any stage can be returned to the storage locations from which the input data for that stage were extracted. Thus only n storage locations are required to perform a transformation of order n.

The operations of Fig. 1 are the addition and subtraction of elements spaced a fixed number of elements apart in the vector. Each stage in the algorithm can be represented by a generalised stage which differs only in the number of elements separating the operands. It is also possible to arrange the addition, subtraction and storage functions in such a way that the only storage required is in the form of serial shift registers. In the design of a real-time Walsh-Hadamard transformer, both of these features simplify the logic required.

Fig. 2 shows a block schematic of a single stage of a real-time transformer,* with an adder, a subtractor, a shift register and a dual data selector. A complete transformer is made up from $p = \log_2 n$ of these stages, r taking the values 1, 2, . . ., (p−1), p. In this circuit, the shift register has two functions. At first, it is used to delay the input to the stage, so that the words forming the first half of the

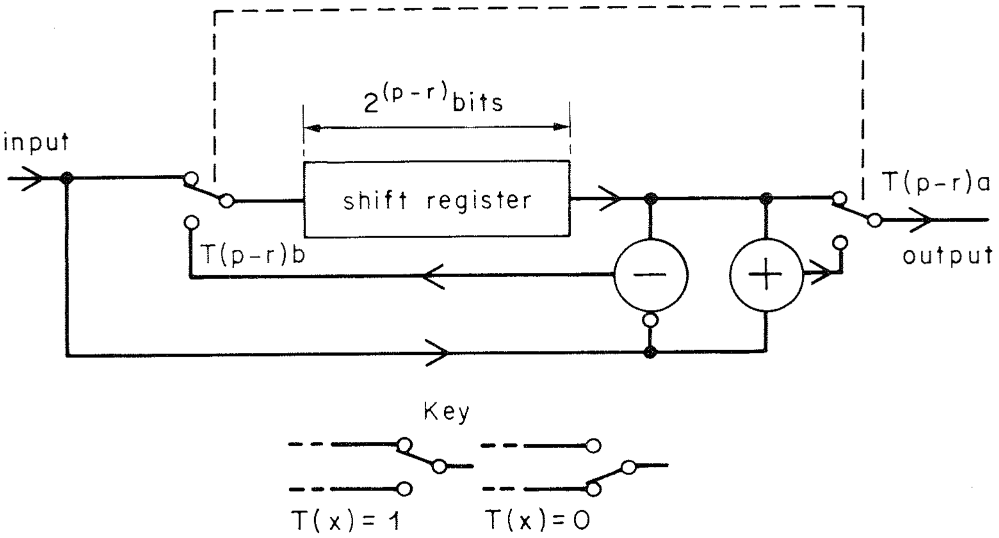

Fig. 2 - Block diagram of r^{th} stage of real-time transformer

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* This arrangement was devised by J.P. Chambers. British Patent Applications 32837/71, 37010/73.

TABLE 1

Block N − 1

Clock Pulse No.	Control T2,T1,T0	Input Vector	1st stage output	2nd stage output	3rd stage output
0	111	A			
1	110	B			
2	101	C			
3	100	D	d−h		
4	011	E	A+E		
5	010	F	B+F	(b−f) − (d−h)	
6	001	G	C+G	(A+E) + (C+G)	((a−c) − (c−g)) − ((b−f) − (d−h))
7	000	H	D+H	(B+F) + (D+H)	((A+E) + (C+G)) + ((B+F) + (D+H))
8	111	a	A−E	(A+E) − (C+G)	((A+E) + (C+G)) − ((B+F) + (D+H))
9	110		B−F	(B+F) − (D+H)	((A+E) − (C+G)) + ((B+F) − (D+H))
10	101		C−G	(A−E) + (C−G)	((A+E) − (C+G)) − ((B+F) − (D+H))
11	100		D−H	(B−F) + (D−H)	((A−E) + (C−G)) + ((B−F) + (D−H))
12	011		a+e	(A−E) − (C−G)	((A−E) + (C−G)) − ((B−F) + (D−H))
13	010			(B−F) − (D−H)	((A−E) − (C−G)) + ((B−F) − (D−H))
14	001			(a+e) + (c+g)	((A−E) − (C−G)) − ((B−F) − (D−H))
15	000				((a+e) + (c+g)) + ((b+f) + (d+h))

Block N + 1

block are available at the inputs to the adder and subtractor at the same time as the words in the second half. During the second half of the process the adder and subtractor simultaneously produce the sums and differences of the elements word by word. The data selector T(p−r)a directs the sums to the stage output and T(p−r)b directs the differences to the shift register input. When all the sums have been clocked out, T(p−r)a then changes over to direct the differences stored in the shift register to the output. At the same time T(p−r)b loads the shift register with the data from the next block. Table 1 shows this pattern for a transform of order 8 for which 3 stages are required, with shift register lengths of 4, 2 and 1 bits respectively. It can be seen that, with this arrangement, the transformation can be done as a continuous process, the results of transforming one input vector following on immediately from the previous result.

The control functions for a transform of order n can be seen, from Table 1, to be the Rademacher functions from the set of Walsh functions, Wal(1,n) for the first stage and Wal(n−1,n) for the final stage.

Each of such stages in a transformer is composed of two distinct parts, the arithmetic and data-selection logic, (which is identical in every stage), and the storage array (which is of a different length in each stage). By constructing each from two printed circuit boards partitioned in this way, a significant saving in the amount of board and logic design can be achieved. To reduce the clock frequency, the transformer operates on parallel data throughout. Fig. 2 therefore represents the processing applied to one bit in one stage and must be repeated as necessary at each stage, and the system clock rate is equal to the input p.c.m. word rate (approximately 13·3 MHz). By making up the stages from smaller boards each of which processes a small number of bits of each word, a modular system can be used, in which each stage is made up to the required word-length by using a number of standard boards.

The number of bits allocated to each word in the fast transformer must increase by one at each stage. This is because the result obtained by adding or subtracting two m-digit numbers contains $m + 1$ significant bits. In general, a transformer of order n contains $p = \log_2 n$ stages. With m-bit numbers applied to the input, the output has $m + p$ bits per word.

The complexity of a particular size of Walsh-Hadamard transformer can be stated relatively simply in general terms because of the similarity of the stages of a fast transformer. To transform n words each of m bits requires p transformer stages where $p = \log_2 n$. Each stage contains a shift register, an adder, a subtractor and two data selectors. In the rth stage, the shift register contains $2^{(p-r)}$ words each of $(m+r)$ bits, the adder and the subtractor each accept inputs of $(m+r - 1)$ bits and produce outputs of $(m+r)$ bits, and the data selectors work with $(m+r)$ bits. This analysis shows that the complexity of a transformer as a function of the number of words per block transformed has two components, the processing part which increases approximately linearly with the number of stages, and the storage which doubles with each additional stage. For transformers with five or less stages, the arithmetic processing forms the major part of the cost, whereas for six or more stages the storage dominates the cost.

3.3. Inverse transformation

It has been shown in Equation (4) that the inverse transformation for symmetric Hadamard matrices involves the same matrix multiplication as the forward transformation. However, the most economical method of fast inverse transformation uses multiplication by the set of sparse matrices in the reverse sequence to that used for the forward transformation. This ensures that the minimum number of bits have to be allocated at each stage in the inverse transformer. This will be demonstrated by continuing the eight-word transformation example of Section 3.1.

For the fast inverse transformation the Walsh-Hadamard matrix of order 8 is expressed as the following product of the sparse matrices S_1, S_2 and S_3 which have been given previously:

$$H_8 = S_1 S_2 S_3$$

In this equation the sparse matrices are multiplied together in the reverse sequence to that shown in Equation (7). Thus the first step in the inverse transformation is to multiply the vector of transform coefficients $\bar{c}$ by the sparse matrix S_3 to produce a vector $\bar{d}$:

$$\bar{d} = S_3 \bar{c}$$

This equation may be re-written as follows:

$$\bar{d} = S_3 S_3 \bar{b} \qquad \text{using (8)}$$

Since

$$S_3 S_3 = 2I$$

$$\bar{d} = 2\bar{b}$$

Because the vector $\bar{d}$ is linearly related to $\bar{b}$, there are the same number of possible values for $\bar{d}$ as for $\bar{b}$. Therefore, every possible value of an element of $\bar{d}$ may be represented with the same number of significant bits as $\bar{b}$. If the sparse matrices are applied in the same sequence as in the forward transformation so that $\bar{c}$ is multiplied by S_1, then there is no simple relationship between the vector produced and $\bar{b}$, and an extra bit must be added to accommodate the increased range of numbers possible.

Similarly, the vectors $\bar{e}$ and $\bar{f}$ produced by the remaining stages of the inverse transformer are related to vectors in the forward transformer by the equations:

$$\bar{e} = 4\bar{a}$$

and

$$\bar{f} = 8\bar{x}$$

As shown in Equation (5), the product of the Hadamard matrix and the vector of coefficients $\bar{c}$ must be divided by n, in this case 8, to reproduce the exact values of the input vector $\bar{x}$. Since n is always an integral power of two for the Walsh-Hadamard transformation, the division is accomplished by relabelling the binary digits with lower significance and no processing is necessary.

The changes in the number of bits required at each stage for the general case of an nth order transformer and inverse transformer with an m-bit input is shown in Fig. 3. For the inverse transformer two versions are compared: the continuous outline shows the number of bits needed to reconstruct the intermediate outputs of the forward transformer; the dashed outline shows the extra bits required for any combination of input values to the inverse transformer. Given that it is practically advantageous to reverse the sequence of the operations in the inverse transformer, two choices remain for the organisation of the transform and inverse transform system. A significant saving in cost can be achieved by choosing that the longest shift registers are used where the number of bits per word is smallest, that is, in the first stage of the transformer and the last stage of the inverse transformer, and the shortest shift registers are used where the word length is greatest.

Bit-rate reduction of the transform coefficients may produce combinations of coefficients which do not transform back to the same numbers that were applied to the input of the transformer. This can result in numbers in the inverse transformer that fall outside the ranges indicated by the continuous outline in Fig. 3. However, it is expected that the errors produced in this way would be small compared with those produced by the bit-rate reduction so that the area bounded by the continuous lines would be sufficient for a practical inverse transformer.

3.4. Experimental real-time Walsh-Hadamard transformer

An experimental real-time transform system, of order 32,[1] has been developed. The input sub-picture, or block, formed by the samples was one-dimensional, that is, the 32

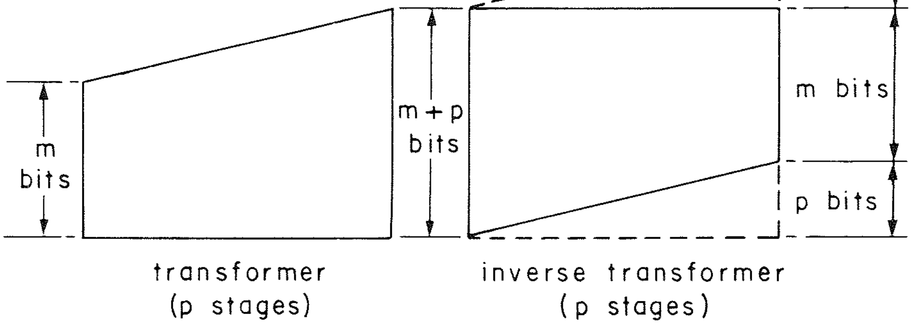

Fig. 3 - Allocation of significant bits in a transform system ($p = \log_2 n$)

samples were taken consecutively from one television line. As the process was a continuous one, the input sub-pictures were immediately adjacent to each other. The system contained, in addition to the forward and inverse transformers, a data processing unit. This enabled the data in the transform domain to be controlled by an external programme, thus permitting a number of bit-rate reduction methods to be applied and their effects on the final output to be observed directly. The p.c.m. input word length was 8 bits and the transform domain word length was 13 bits, and the length of the word from the output of the inverse transformer was limited to 8 bits by the digital to analogue converter. This apparatus showed that it is entirely feasible to carry out real-time transformation of broadcast quality television pictures at an input data rate of 106 Mbits per second.

4. Walsh spectrum analyser

Equation (6) shows that a coefficient of the transformation may be calculated by cross-correlating the appropriate Walsh function and the input signal. Thus, a practical Walsh spectrum analyser consists of a correlator controlled by a Walsh function generator.

A spectrum analyser suitable for use with digital television signals has been constructed to measure the maximum amplitudes of the transform coefficients.[2] The Walsh function generator used in the equipment is based on 6-bit binary-rate-multiplier integrated circuits and produces all functions of a given period in order of increasing sequency. Walsh functions with several different periods could be generated, so that the spectrum for different sizes of transformation could be measured. Further, by suitably controlling the generator, the functions could be correlated with signals corresponding to rectangular areas of the picture, so that the benefits of two-dimensional transforms could be assessed. A store was incorporated to hold the maximum values of the coefficients. Thus, as each coefficient was measured, it was compared with the stored value already measured for that Walsh function, and the larger value of the two retained in the store.

5. Bit-rate reduction in the transform domain

Any digital transformation which involves arithmetic operations inevitably produces a larger number of bits in the words describing the transform than were used in the linear p.c.m. domain. It has been shown that a Walsh-Hadamard transformer requires $m + \log_2 n$ bits to represent each coefficient of the transform precisely, where m is the number of bits in the p.c.m. input words and n is the order of the transformation. For example, for the transformation of a block of 64 8-bit words, the output must be represented by $8 + \log_2 64$, that is, 14 bits per coefficient.

The number of bits needed to represent the transform may be reduced considerably from this theoretical value without noticeably affecting the picture quality. This can be achieved partially by removing some of the most significant bits from the coefficients, because television signals, whilst being theoretically capable of doing so, very seldom use the full range of values of each coefficient, except of course for the Wal(0,n) coefficient which represents the mean level of the sub-picture. It is also possible to remove some of the least significant bits in all the coefficients as the signal-energy contained therein is negligibly small.

The number of least significant bits that may be omitted can be estimated by considering the amount of energy represented by the bits before and after transformation. According to Parseval's Theorem[6] energy is conserved in an orthonormal transformation. So, to consider the equivalence of energy between the input signal and the transformed signal, the transformed coefficients must be divided by the normalising factor $n^{½}$. This represents a shift of significance of the binary digits of the coefficients downwards by ½ $\log_2 n$ bits. In Fig. 4 the binary digits representing the coefficients for different sizes of transformation are depicted as boxes; digits representing equal amounts of energy are shown at the same horizontal level.

For each size of transformation, ½ $\log_2 n$ least significant bits of each coefficient represent amounts of energy smaller than that described by the least significant bit of the 8-bit p.c.m. input signal ($\log_2 n = 0$). Omitting these bits in the transform will introduce an average noise energy less significant than that due to the 8-bit linear p.c.m. quantisation. For odd values of $\log_2 n$ the average reduction may

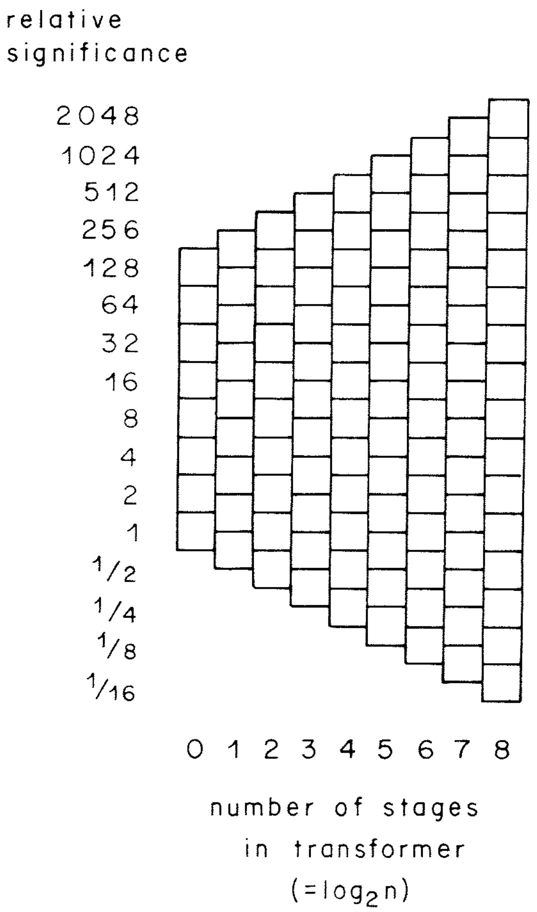

Fig. 4 - Relative significance of bits in the transform domain for different orders of transformation

be made fractional by omitting more bits from some coefficients than from others.

For both these forms of bit-rate reduction, the least possible picture degradation results when using the nearest approximation to the correct value, whenever the correct value cannot be used. The approximation is obtained by rounding the coefficient to compensate for the loss of the lower significant bits and by limiting to a maximum permissible value whenever the transform coefficient exceeds this value.

5.1. Results obtained using real-time transformer

The output pictures produced by the real-time transformer, when the simple bit-rate reduction techniques described above were applied, shows a slight degradation, approximately equivalent to ½ bit per sample, when compared with those produced by a linear p.c.m. system using the same number of bits per sample and an appropriate amount of 'dither'.* This difference remained relatively constant over a range of 5 bits per sample to 8 bits per sample. With 5 bits per sample or less, or without the addition of the dither signal, the effects produced by the Walsh-Hadamard transformation and by linear p.c.m. were of different characters, and comparisons made between the two systems under these conditions were, therefore, difficult to make.

To enable all normal broadcast colour pictures to be represented adequately, it was found that an average of two most significant bits could be omitted from the transform signal. Individual pictures and the majority of normal pictures could still be represented adequately even if more than the two most significant bits were omitted, but a restriction on the set of pictures which can be represented is clearly not possible in broadcast applications. In order to achieve a mean bit rate of k bits per sample, it would be necessary to omit a further 13 – (2+k) bits from the lower significant end of the words describing the transform coefficients. It is this coarse quantisation and rounding of the transform coefficients which give rise to the increase in the visible noise in the final picture and the consequent degradation of the picture quality. Where fractional mean bit rates are required, a greater number of lower significant bits must be omitted from some coefficients than from others. In these cases, it was found to be preferable to omit more bits from the coefficients of higher sequency than from those of lower sequency. The resulting increase in high sequency noise was found to be subjectively less objectionable than the corresponding increase in low sequency noise.

5.2. Extension to other transform orders

The results obtained with the real-time transformer were extended to other sub-picture sizes and shapes by carrying out measurements on the Walsh spectrum of the signals. By finding changes in the Walsh spectrum for different sizes of transformation, the changes in the required bit rate were estimated.

It has been shown that $\frac{1}{2}\log_2 n$ least significant bits may be omitted from each coefficient while maintaining picture quality. Therefore, a further reduction of more than $\frac{1}{2}\log_2 n$ bits is required before the data rate of the transformed signal is lower than that of the input signal.

The number of most significant bits which can be removed was found by measuring the maximum amplitudes of the coefficients using a Walsh spectrum analyser. It was found that signals containing a number of large amplitude black-to-white and white-to-black transitions, and signals containing large amplitude colour subcarrier produced the largest coefficients. Such signals would be produced for pictures containing alphanumeric characters and areas of highly saturated colour. In the following results, a combined spectrum describing the maximum amplitudes required for these two types of signal is used.

The spectrum of maximum amplitudes has been measured for transformations of 8, 16, 32 and 64 signal samples, and has been measured for both linear and two-dimensional sub-pictures. In each case, the average number of most significant bits to be omitted from each coefficient has been measured and subtracted from the $8 + \frac{1}{2}\log_2 n$ bits which remain after the redundant least significant bits have been omitted. Hence, for these methods of bit-rate reduction, Table 2 shows the average number of bits per coefficient needed to represent the transformed signal without noticeable loss of quality. The results are arranged in terms of the block size and a block shape factor, which is the ratio of the number of samples in a horizontal line in the block to the number of samples in a vertical line in the block. For example, for a block size of 32 arranged as 4 samples horizontally and 8 samples vertically, the block shape factor is ½.

TABLE 2

		Block Shape										
		$^1/_{16}$	$^1/_8$	$^1/_4$	$^1/_2$	1	2	4	8	16	32	64
	8				8·88		8·75		8·63			
Block	16			9·00		8·88		8·81		8·63		
size	32		8·88		8·81		8·94		8·94		8·81	
	64	8·77		8·75		8·66		8·94		8·95		8·89

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* Dither: the deliberate addition of random or pseudo-random noise or other disturbance to the analogue signal before digital conversion, the objective being to reduce the visible effects of quantisation.

Thus the number of bits per coefficient necessary to produce pictures similar in quality to 8-bit p.c.m. for various block sizes and block shapes range from 8·63 to 9·00. This shows that it is not possible to reduce the bit rate of Walsh-Hadamard transformed signals just by restricting the range of the transform coefficients. Further, the results show that for this bit-rate reduction method there is no definite advantage in transforming two-dimensional blocks of picture. Therefore, more efficient methods of coding the transform coefficients, such as companding or adaptive coding must be found if any effective reduction in bit-rate is to be made with Walsh-Hadamard transformation.

With a non-linear coding method, the large amplitude errors, which would sometimes occur with simple limiting of the coefficient maximum values, are shifted in significance to become small errors of a magnitude comparable with the least or the second least significant bit. This reduction of significance would permit these errors to occur much more frequently before the same picture impairment became visible, thus the mean bit rate might be reduced by about 1 bit per sample.

This preliminary conclusion arises from tests using high quality television signals. Work carried out to date on d.p.c.m. with non-linear quantising[7] as an alternative means of bit-rate reduction indicates that the two approaches yield broadly similar results. However, further work may be necessary to establish which of these approaches is preferable when the 'bit-rate reduced' digital signal is subject to errors.

An adaptive system, which effectively adjusts the transmission system to an optimum for each individual picture or sub-picture would, in principle, greatly reduce the bit rate. Such systems are, however, usually prone to overload failure if the reduction in the number of bits per coefficient is appreciable and failures of this nature would be unacceptable for broadcast applications.

There are two reasons for the difference between the results obtained from spectral analysis for a one-dimensional block of 32 samples (8·81 bits per sample) and those obtained from the experimental real-time transformer (8·0 bits per sample). First, 3 significant bits were omitted from the lower end of the coefficient words in the tests using the transformer instead of the 2·5 indicated by theory; this slightly degraded the picture quality. Secondly, the spectrum analyser recorded electronically the number of bits required to represent the transform coefficients without over-ranging errors. With the real-time transformer it was found that some over-ranging could be tolerated because of the low visibility of relatively large and infrequent errors. Thus, it was possible to omit a greater number of most significant bits (approximately ¼ bit per sample).

6. Conclusions

It has been shown that real-time Walsh-Hadamard transformation of broadcast-quality television pictures is entirely feasible. Using a modular system a transformer can be constructed of any size in terms of both transform order and input word length. Although small orders of transform can be constructed simply and cheaply, larger transformers become increasingly more expensive, as large numbers of storage elements are required.

Two simple methods of bit-rate reduction, rounding and limiting of the transform coefficients, can reduce the bit rate in the transform domain to be the same as in the original p.c.m. signal with only a small impairment of the picture, if no limitation is to be imposed on the input pictures. No significant differences were found, when using rounding and limiting, between the results for different block sizes or shapes. To reduce the bit rate further requires more complex methods of bit-rate reduction such as non-linear coding of the transform coefficients or an adaptive system. With both of these methods, it is likely that more advantage would be gained by transforming larger blocks of data.

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